Section 4.5
Linearization and Newton's Method

Linear Approximation - using the tangent to approximate the function

If f is differentiable at $x = a$, then the equation of the tangent line,

$$L(x) = f(a) + f'(a)(x - a)$$

defines the linearization of f at a.

The approximation $f(x)$ is approximately $L(x)$

The point $x = a$ is the CENTER of the approximation

Find the linearization of $f(x) = \sqrt{1 + x}$ at $x = 0$, and use it to approximate $\sqrt{1.02}$ without a calculator.

Use linearization to approximate at x = 2 for

$$f(x) = x^2 - \frac{1}{x}$$

Find the linearization of $f(x) = \cos x$ at $x = \pi/2$ and use it to approximate $\cos 1.75$ without a calculator.

Differentials

Let $y = f(x)$ be a differentiable function.
The differential dx is an independent variable.

The differential dy is

$$dy = f'(x)\, dx$$

Find dy if $f(x) = 3x^2 - 2x$ and $x = 4$ and $dx = .2$

$f'(x) = 6x - 2$

$dy = (6x - 2)\,dx$

$dy = (22)(.2) = 4.4$

Find the differential dy and evaluate dy for the given values of x and dx.

a.) $y = x^5 + 37x, \quad x=1, \quad dx = 0.01$

Find the differential dy and evaluate dy for the given values of x and dx.

b.) $y = \sin 3x$, $x = \pi$, $dx = -.02$

$y' = 3\cos 3x$

$dy = 3\cos(3\pi)\cdot(-.02)$

$= -3\cdot(-.02)$

$= .06$

Find the differential dy and evaluate dy for the given values of x and dx.

$2+y=2y$

c.) $x + y = xy, \qquad x = 2, \qquad dx = 0.05$

$$1 + 1\frac{dy}{dx} = 1y + x\frac{dy}{dx}$$

$$\frac{dy}{dx} - x\frac{dy}{dx} = y - 1$$

$$\frac{dy}{dx}(1-x) = y-1$$

$$\frac{dy}{dx} = \frac{y-1}{1-x}$$

$$dy = f'(x)\,dx$$

$$dy = \left(\frac{y-1}{1-x}\right)dx$$

$$dy = \frac{2-1}{1-2}\cdot(.05)$$

$$= \frac{1}{-1}\cdot(.05)$$

$$= -.05$$

Find the change in volume of a sphere when the radius is increased from 3 to 3.1 inches

$$V = \frac{4}{3}\pi r^3$$

@ $3 \rightarrow 113.097$

@ $3.1 \rightarrow 124.788$

11.69

Similar

$$V' = 4\pi r^2$$

$$dV = V' \cdot dr$$

$$dV = 4\pi r^2 \cdot dr$$

$$dV = 4\pi (3)^2 \cdot (.1) = 11.310$$

The radius r of a circle increases from r = 10 m to 10.1 m. Use dA to estimate the increase in the circle's area A.

$A = \pi r^2$

$A' = 2\pi r$

$dA = A' \cdot dr$

$dA = 2\pi r\, dr$

$dA = 2\pi(10)(.1)$

$= 2\pi$

Inflating a bicycle tire changes its radius from 12 inches to 13 inches. Use differentials to estimate the change in the perimeter of the tire.

$P = 2\pi r$

$P' = 2\pi$

$dP = P' \cdot dr$

$dP = 2\pi \cdot 1$

$dP = 2\pi$

In the late 1830s, the French phsyiologist Jean Poiseuille discovered the formula we use today to predict how much the radius of a partially clogged artery has to be expanded to restore normal flow. His formula was

$$V = k r^4$$

says that the volume V of fluid flowing through a small pipe or tube in a unit of time at a fixed pressure is a constant times the fourth power of the tube's radius r. How will a 10% increase in r affect the V?